B010

3D Finite-Difference Modeling of Borehole Wave Propagation

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SUMMARY

Finite-difference (FD) method for 3D simulation of sonic waves propagating in a borehole and surrounding 3D heterogeneous elastic medium is presented. It is based on explicit second-order staggered grid FD scheme that solves the first-order elastic wave equations in cylindrical coordinates. Special modification of Perfectly Matched Layer (PML) for cylindrical coordinate system is developed and implemented. Essential 3D nature of the waves processes for realistic models claims necessity to use parallel computations. Parallelization is performed on the base of domain decomposition approach and implemented under Message Passing Interface (MPI). Result of numerical experiments is presented.

Introduction. Full wave acoustic logs are very important borehole measurements providing knowledge about physical properties of surrounding rocks. Historically these methods have been based on the use of axially symmetric, or monopole, wave phenomena in a fluid-filled borehole. Recently, logging tools based on excitation and reception of nonaxially symmetric wave phenomena have been developed and used in order to explore near borehole media. Moreover, real media themselves are 3D heterogeneous, so real sonic wavefields never possess any axial symmetry.

In order to be able to recover physical properties of surrounding rocks by sonic data one should fully appreciate key peculiarities of elastic waves propagation through borehole imbedded inside 3D heterogeneous elastic formations. We believe that the most effective way to do that is implementation of a representative series of numerical experiments. This follows necessity to develop algorithms and to create software providing a person with a possibility to perform a range of simulations for a set of models and source positions. Due to the large numbers of unknowns involved in a 3D elastic wave problem, the FD approach offers some advantages in comparison with others methods: it is easy and straightforward for implementation, can be parallelized in a simple and natural way and its RAM requirements is O(N) with N being the total number of unknowns. In its own turn the total number of unknowns depends on a number of grid nodes needed for required quality of finite-difference approximation. Most of the previous 3D FD studies are done for Cartesian coordinates (see, for example, [1]). But, the use of these coordinates leads to step-like representation of the sharpest interface of the problem - interface between fluid-filled borehole and enclosing rocks - and produces rather strong artificial numerical scattering. In order to reduce this artifact one should use spatial grid steps being very small in comparison with borehole radius and, so, comes to huge RAM demands. In our opinion, the best way to approximate this interface is to use cylindrical coordinates. Following this way we could restrict RAM claims down to 30Gb $\div$ 40Gb for commonly used sizes of a model and frequency range 5kHz $\div$ 25kHz.

Statement of the Problem. Sonic waves propagation in heterogeneous elastic media is governed by initial-boundary value problem for the *t*-hyperbolic first-order system of partial differential equations for velocity vector $\vec{u} = (u_r, u_\varphi, u_z)$ and stress "vector" $\vec{\sigma} = (\sigma_{rr}, \sigma_{\varphi\varphi}, \sigma_{zz}, \sigma_{r\varphi}, \sigma_{\varphi z}, \sigma_{rz})$. Specific feature of the problem is the presence of liquid-filled circular tube with interface at $r = R$. Sonic source is supposed to be located within the liquid and currently is simulated as volumetric point source. This leads to the following statement of the problem:

$$\rho \frac{\partial \vec{u}}{\partial t} = A \frac{\partial \vec{\sigma}}{\partial r} + \frac{1}{r} B \frac{\partial \vec{\sigma}}{\partial \varphi} + C \frac{\partial \vec{\sigma}}{\partial z} + \frac{1}{r}(A - D)\vec{\sigma};$$

$$M \frac{\partial \vec{\sigma}}{\partial t} = A^T \frac{\partial \vec{u}}{\partial r} + B^T \frac{1}{r} \frac{\partial \vec{u}}{\partial \varphi} + C^T \frac{\partial \vec{u}}{\partial z} + \frac{1}{r} D^T \vec{u} + \vec{F}(r, \varphi, z, t);$$

$$F(r, \varphi, z, t) = \frac{F(t)}{3\lambda + 2\mu} \frac{\delta(r - r_0, \varphi - \varphi_0, z - z_0)}{2\pi r} (1,1,1,0,0,0)^T$$

$$\vec{u}\big|_{t=0} = \vec{\sigma}\big|_{t=0} = 0; \quad [\vec{u}]_{r=R} = 0; \quad [\sigma_{rr}]_{r=R} = [\sigma_{r\varphi}]_{r=R} = [\sigma_{rz}]_{r=R} = 0$$

A, B, C, D here are 3x6 matrices, M is 6x6 matrix ([2]). Discretization of this system is performed by means of Virieux's-like staggered grid in space and time and leads to well-studied explicit finite difference scheme ([3]).

The main trouble the person faces on when dealing with FD scheme in cylindrical coordinate system is necessity to perform periodically azimuth grid refinement in order to avoid too large azimuth step followed by poor spatial approximation that finally will entail rather strong numerical dispersion. In order to avoid this trouble one should take very fine grid step in the vicinity of $r = 0$, but this will claim too small time step due to the Courant's condition and, so, will be finalized in very high time-consuming requirements. So, the best way is to refine azimuth step from time to time in order to provide uniform spatial approximation. We choose the following way of refinement: azimuth step h_φ is two times decreased as radial distance is doubled. Every time, in order to move from sparse to fine grid one should interpolate velocity vector and stress components. Different components are situated at the different grid points as one can see on the Fig.1. In order to provide the uniform approach to both situations Fourier transform based interpolation is chosen. We believe it is the best one interpolation procedure for this specific situation as it provides exponential accuracy due to 2π-periodicity of all functions involved with respect to azimuth angle.

Perfectly Matched Layers for Cylindrical Coordinates. Let us introduce PML with respect to radius *r* only as with respect to *z* it can be introduced in a usual way ([4]). In order to do that one should in a usual way split vectors $\vec{u}$ and $\vec{\sigma}$ onto parallel and perpendicular components: $\vec{u} = \vec{u}^\perp + \vec{u}^\|$, $\vec{\sigma} = \vec{\sigma}^\perp + \vec{\sigma}^\|$. After that some manipulations ([5]) lead to the following version of PML elastic wave equations for cylindrical coordinates (let us remind that target area is within the circle $r \le R_0$, while PML fills the ring $R_0 \le r \le R_1$):

$$\frac{\partial \vec{u}^\|}{\partial t} = C\frac{\partial \vec{\sigma}}{\partial z};\quad \left(\frac{\partial}{\partial t} + \alpha(r)\right)\rho\vec{u}^\perp = A\frac{\partial \vec{\sigma}}{\partial r} + \frac{1}{r}B\frac{\partial \vec{\tilde{\sigma}}}{\partial \varphi} + (A-D)\frac{\vec{\tilde{\sigma}}}{r}$$
$$M\frac{\partial \vec{\sigma}^\|}{\partial t} = C^T\frac{\partial \vec{u}}{\partial z};\quad \left(\frac{\partial}{\partial t} + \alpha(r)\right)M\vec{\sigma}^\perp = A^T\frac{\partial \vec{u}}{\partial r} + \frac{1}{r}B^T\frac{\partial \vec{\tilde{u}}}{\partial \varphi} + D^T\frac{\vec{\tilde{u}}}{r} \quad (1)$$

Damping factor $\alpha(r) \ge 0$, new vectors $\vec{\tilde{u}}$ and $\vec{\tilde{\sigma}}$ satisfy to the following system of ordinary differential equations:

$$\frac{\partial \vec{\tilde{u}}}{\partial t} + \frac{\beta(r)}{r}\vec{\tilde{u}} = \frac{\partial \vec{u}}{\partial t} + \alpha(r)\vec{u};\quad \frac{\partial \vec{\tilde{\sigma}}}{\partial t} + \frac{\beta(r)}{r}\vec{\tilde{\sigma}} = \frac{\vec{\partial} u}{\partial t} + \alpha(r)\vec{u};\quad \beta(r) = \int_{R_0}^{R_1}\alpha(\xi)d\xi \quad (2)$$

As is proven in [5], in time frequency domain (1) and (2) are equivalent to transformation of real variable r to complex one $\tilde{r} = r + i\beta(r)/\omega$ and, so, provides exponential decay of "waves" within PML area when $r \to \infty$.

We would like to pay the attention on the main advantage of the proposed PML – one should split variables within PML with respect to *z* and *r* only, there is no splitting with respect to azimuth.

Parallel implementation. Parallel computations are implemented with the help of domain decomposition. Under this approach the total 3D model is sliced into a number of disc-like subdomains and each subdomain Ω_i is assigned to a separate processor element. Waves travelling in the model pass different domains, which requires communication between neighboring processors. This approach is very similar to the one presented in [6] for simulation of 2D elastic waves propagation. The difference, besides essential 3D nature of the problem, is that now these subdomains are not overlapping. This becomes possible thanks to the use of MPI non-blocking functions Isend and Ireceive.

Numerical experiments. The series of numerical experiments have been implemented for a range of source frequencies, positions and models of surrounding elastic media. Particular attention was paid to the problem with eccentric source as being the most interesting for practical reasons. The main question is: how strong does disturb the total wave field when source position is essentially eccentric? One can estimate this disturbance by comparison Fig.2 and Fug.3. The only difference of these experiments is in source position. For both of them the same model is used: borehole with diameter of 0.2 m filled with a liquid with wave propagation velocity $V_f = 1524$ m/sec and density $\rho_f = 1000$ kg/m^3 is imbedded within homogeneous elastic medium with wave propagation velocities $V_p = 3500$ m/sec, $V_s = 2000$ m/sec and density $\rho_f = 2300$ kg/m^3. For both experiments Ricker's impulse with dominant frequency of 10 kHz is used for volumetric point source. On Fig.1 one can follow propagation of σ_{rr} radiating by this source placed at the origin (axial symmetry statement), while Fig.2 represents propagation of the same stress component, but generating by eccentric source position (eccentricity is taken equal to one quarter of bore hole diameter).

Both series of snapshots possess evident presence of the most intensive group of surface/tube waves, but there are rather strong difference in their behavior - the wave field generating by eccentric source is zigzag-like nature of propagation. So, one can conclude, that eccentric source generated essentially 3D wave fields and structure of these wave fields should be taken into account in order to do reliable data processing and interpretation.

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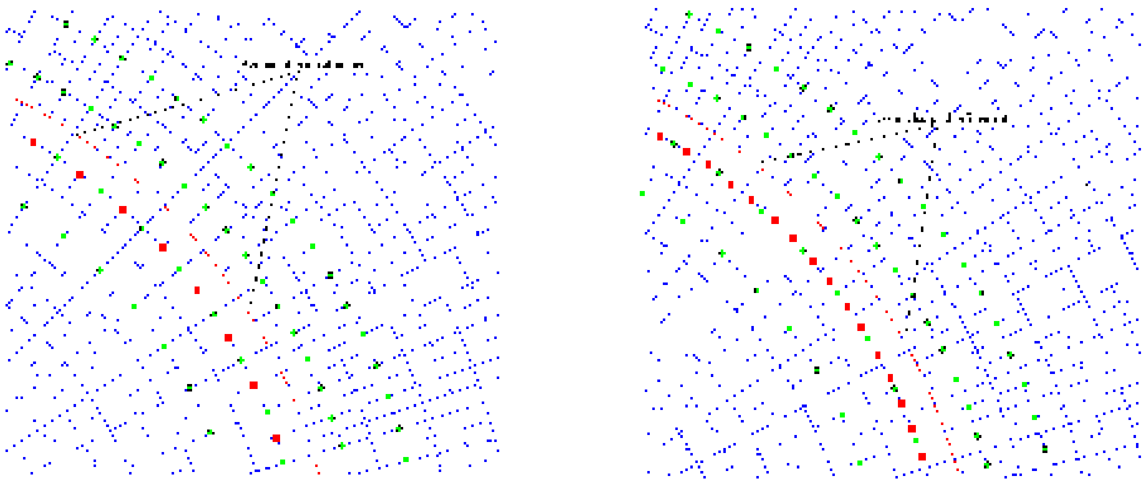

Fig.1. Polar grid refinement. Red circle – line of refinement. Green circles – known values. Red diamonds – values to be interpolated. Left - $\sigma_{rr}, \sigma_{\varphi\varphi}, \sigma_{zz}, \sigma_{rz}, u_{\varphi}$, right - $\sigma_{r\varphi}$.

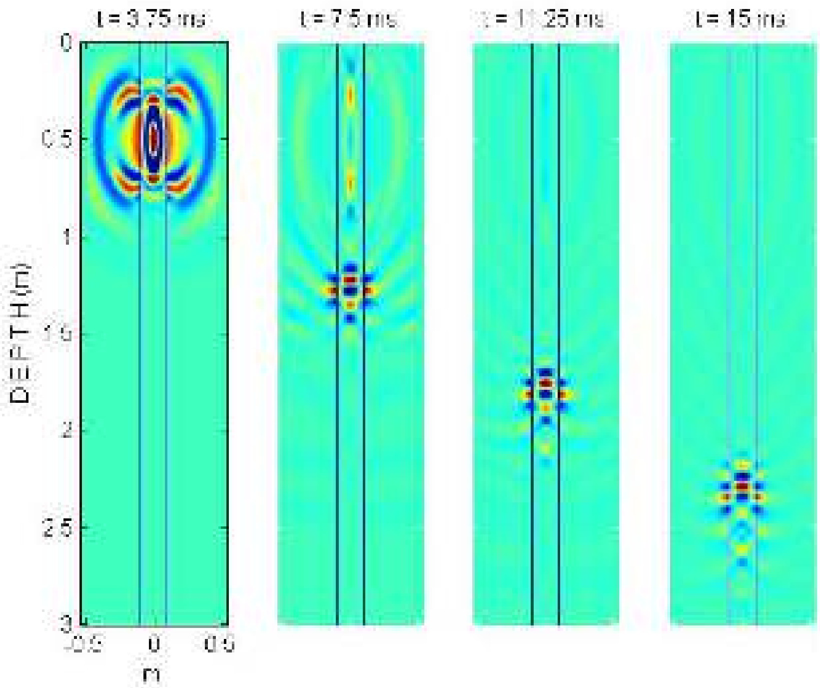

Fig.2. Axial symmetry volumetric source position. Propagation of σ_{rr} within the plane $\varphi = 0, \varphi = \pi$. Two vertical lines represent borehole projection onto this plane

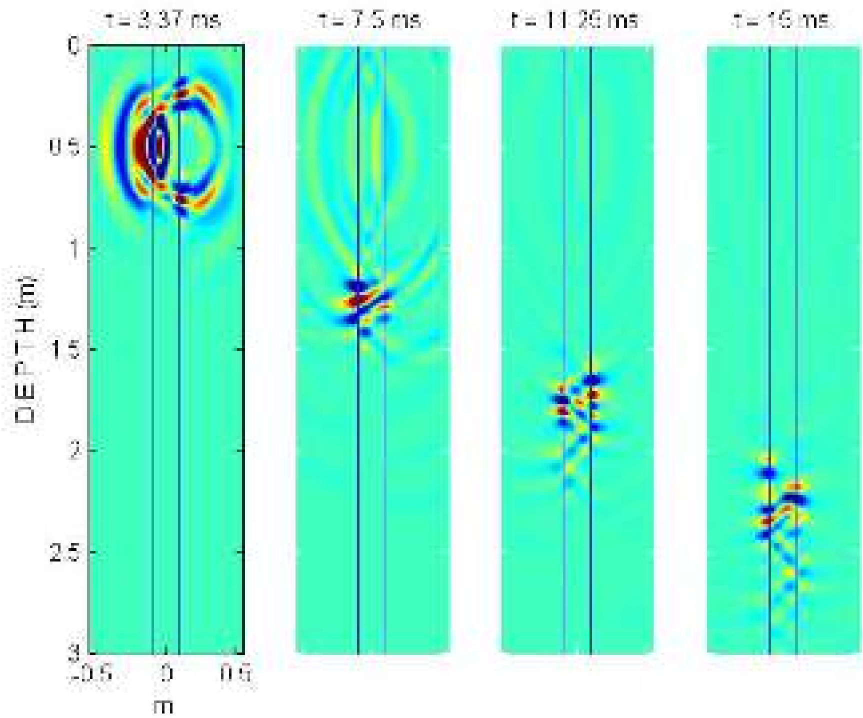

Fig.3. Eccentric volumetric source position. Propagation of σ_{rr} within the plane $\varphi = 0, \varphi = \pi$. Two vertical lines represent borehole projection onto this plane